Parallel Session

Population Biology II

INTERACTIONS OF TRAVELING WAVES IN THE CONTEXT OF POPULATION GENETICS

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Keywords: Population genetics, Traveling waves.

Evolution is driven by forces that constantly reshape the genetic distribution observed in a population. Selection occurs on the genotypes of individuals within a single population present in a given environment; while dispersal of individuals in this environment takes place. In the case of several loci for a sexual population, recombination between loci introduces a new layer of complexity.

Here we investigate a two-locus two-allele model which accounts for selection, dispersal through diffusion in space (homogeneous and heterogeneous spaces will be explored) and recombination with an arbitrary rate, in order to determine the interaction between loci under selective pressures. A system of 3 coupled PDEs is established and studied.

Classical behaviors of solutions in the case of a single Fisher–KPP PDE (one locus) pertain to the family of traveling waves or stationary solutions (clines). In the case of two loci, interactions between the waves can occur. This phenomenon is studied using generalized traveling waves. Different kind of interactions are shown, and predictions of the outcome are given.

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TWO-LOCUS CLINES MAINTAINED BY DIFFUSION AND RECOMBINATION IN A HETEROGENEOUS ENVIRONMENT

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Keywords: Population genetics, Parabolic partial differential equations, Migration and selection, Diffusion.

A population-genetic migration-selection model is studied which is continuous in space and time. The model assumes that two diallelic, recombining genetic loci are under spatially varying selection. The habitat is an open, bounded domain in Rn, in which migration occurs by diffusion and is modeled by the Laplacian. The non-constant stationary solutions of the resulting system of semilinear parabolic PDEs are called clines. We investigate existence, uniqueness, and stability of such clines. We prove existence, uniqueness, and stability for limiting cases, such as weak recombination or strong recombination using, among others, perturbation methods as well as persistence theory.

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LARGE DEVIATIONS THEORY WITH APPLICATION TO RARE EVENTS IN BACTERIAL POPULATIONS

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Joint work with R. Azencott (UH), B. Geiger (SMU)

Keywords: Large deviations, Genetic evolution, Bacterial population, Most likely evolutionary path.

Radical shifts in the genetic composition of large cell populations are rare events with quite low probabilities, which direct numerical simulations generally fail to evaluate accurately. We develop an applicable large deviations framework for a class of Markov chains used to model genetic evolution of bacterial populations. We illustrate this framework by computing the most likely evolutionary paths describing emergence of genotypes with lower fitness in realistic parameter settings.

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Parallel Session

Population Biology II

CONSTANT EFFORT HARVESTING MODELS WITH ALLEE EFFECTS IN RANDOMLY VARYING ENVIRONMENTS

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Keywords: Stochastic differential equations, Harvesting models, Allee effects, Random environments.

In a randomly varying environment, we model the growth of the harvested population by a very general Stratonovich stochastic differential equation (SDE)

$$dX(t) = f(X(t))X(t)dt + \sigma X(t)dW(t) - qEX(t)dt,$$

where $X(t)$ is the harvested population size at time t, $f(x)$ is its geometric average *per capita* growth rate (assumed to be of class C^1 with $f(0^+) \neq 0$ finite), $\sigma dW(t)/dt$ describes the effect of environmental random fluctuations on the growth rate ($W(t)$ being a standard Wiener process and $\sigma > 0$ an intensity parameter), E is the constant harvesting effort applied and $q > 0$ is the catchability. Previously ([2], [3], [4]), the existence and uniqueness of the solution and the existence of a stationary density were studied for f strictly decreasing, the usual case where the *per capita* resource availability for survival and reproduction diminishes when the population increases. Here, we will study the case of Allee effects (see [1]) in which one observes a surprising depression (accompanied by its growth) of $f(x)$ when x is small, due, for example, to the difficulty of finding mating partners or setting up effective collective defence mechanisms against predators. With the purpose of obtaining properties (on existence and uniqueness of the solution or on existence of a stationary density) that are robust w.r.t. the choice of the specific model for f (several have been proposed in the literature), we will consider a general function f with Allee effects. Namely, we will only assume that there are constants $L > 0$ and $K > L$ such that $f'(x) > 0$ for $0 < x < L$, $f'(x) < 0$ for $x > L$, and $f(x) < 0$ for $x > K$. We consider two types of Allee effects: strong (when $f(0^+) < 0$) and weak (when $f(0^+) > 0$). Previous results for models without harvesting ([5], [6], [7]) are now extended to models with harvesting. For illustration, we use the particular case of a logistic-like Allee effects model with a parametrization slightly different from the usual one (to allow comparisons with the logistic model), namely $f(x) = r(1 - \frac{x}{K})\frac{x-A}{K-A}$, with an Allee effect strength parameter A ($-K < A < 0$ for weak Allee effects, $0 < A < K$ for strong Allee effects). When $A \to -\infty$, we retrieve the logistic model without Allee effects.

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References

[1] W. C. Allee, A. E. Emerson, O. Park, T. Park, K. P. Schmidt. *Principles of Animal Ecology.* Saunders, Philadelphia, 1949.

[2] C. A. Braumann. (1999). *Variable effort fishing models in random environments.* Mathematical Biosciences 156, 1–19.

[3] C. A. Braumann. (2002). *Variable effort harvesting models in random environments: generalization to density dependent noise intensities.* Mathematical Biosciences 177 & 178, 229–245.

[4] C. A. Braumann. (2007). *Harvesting in a random environment: Itô or Stratonovich calculus.* J. Theoretical Biology 244, 424–432.

[5] C. A. Braumann, C. Carlos. (2013). *Allee effects in randomly varying environments.* Proceedings 2013 International Conference on Computational and Mathematical Methods in Science and Engineering, 304–307.

[6] C. A. Braumann, C. Carlos. (2015). *Weak Allee effects population growth models in a random environment.* Proceedings 60th ISI World Statistics Congres, 2455–2460.

[7] C. Carlos, C. A. Braumann. (2017). *General population growth models with Allee effects in a random environment.* Ecological Complexity 30, 26–33.

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PERFORMANCE OF FISHING POLICIES FOR POPULATIONS WITH WEAK ALLEE EFFECTS IN A RANDOM ENVIRONMENT

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Joint work with Carlos A. Braumann (Departamento de Matemática, Escola de Ciências e Tecnologia, Universidade de Évora) and Clara Carlos (Escola Superior de Tecnologia do Barreiro, Instituto Politécnico de Setúbal), both at: Centro de Investigação em Matemática e Aplicações, Instituto de Investigação e Formação Avançada, Universidade de Évora.

Keywords: Stochastic differential equations, Harvesting models, Allee effects, Profit optimization, Stochastic optimal control.

In a random environment, we describe the growth of a population subjected to harvesting through stochastic differential equations (as in [1, 2]). We assume that the population is under the influence of weak Allee effects, that is, at very low values of population size, we observe lower *per capita* growth rates instead of the higher rates one would expected considering the higher availability of resources per individual. The presence of weak Allee effects when population size is low may be due to the difficulty in finding mating partners or in constructing a strong enough group defence against predators.

We consider the population natural growth to follow a logistic-like model with Allee effects and that the rate of harvesting is proportional to the existing population and to the effort exerted in the capture.

The main goal of this work is to compare the performance of two fishing policies: one with variable effort, here named optimal policy, and the other with constant effort, denoted by sustainable optimal policy. The first results in a fishing effort varying rapidly and abruptly depending on population size which, in a random environment, also varies constantly. This type of policy is inapplicable from the practical point of view. In addition, this policy requires the estimation of population size at each time instant, which is usually an expensive, inaccurate, and time-consuming task. The second policy considers the application of a constant effort over time and predicts the sustainability of the population as well as the existence of a stationary density for its size (see [3]). This policy has the advantage of being applicable, easily implemented and does not require knowledge of population size at any given time. The performance of the two policies will be assessed by the profit obtained over a finite time horizon. A similar study has been done for the logistic model without Allee effects (see [4]), which will allow us to assess the consequences of Allee effects.

Using realistic data based on a fish population, we will quantify the reduction in profit when choosing the optimal sustainable policy with constant effort instead of the optimal and inapplicable policy with variable effort.

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References

[1] J. R. Beddington, R. M. May. (1977). *Harvesting natural populations in a randomly fluctuating environment*, Science 197, pp463–pp465.

[2] C. A. Braumann. (1985). *Stochastic differential equation models of fisheries in an uncertain world: extinction probabilities, optimal fishing effort, and parameter estimation*, Mathematics in Biology and Medicine (V. Capasso, E. Grosso, and S. L. Paveri-Fontana, editors), Springer, Berlin, pp201–pp206.

[3] C. A. Braumann. (2002). *Variable effort harvesting in random environments: generalization to density-dependent noise intensities*, Math. Biosciences 177 & 178, pp229–pp245.

[4] N. M. Brites, C. A. Braumann. (2017). *Fisheries management in random environments: Comparison of harvesting policies for the logistic model*, Fisheries Research 195, pp238–pp246.

Parallel Session

Population Biology II

STRENGTH IN NUMBERS: HOW DEMOGRAPHIC NOISE CAN REVERSE THE DIRECTION OF SELECTION

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Joint work with George Constable (University of Bath), Alan McKane (University of Manchester), Corina Tarnita (Princeton)

Keywords: Evolution, Altruism, Population dynamics, Demographic noise.

Deterministic evolutionary theory robustly predicts that populations displaying altruistic behaviours will be driven to extinction by mutant cheats that absorb common benefits but do not themselves contribute. In this talk I will show how demographic stochasticity can in fact reverse the direction of selection in favour of the cooperative phenotype if their behaviour appreciably alters the carrying capacity of the population. I will present an analysis of a simple but general model of population dynamics in the specific context of public goods production, and derive explicit conditions for stochastic selection reversal. The talk will be based on work published recently in PNAS [1]

References

[1] George W. A. Constable, Tim Rogers, Alan J. McKane and Corina E. Tarnita (2016) *Demographic noise can reverse the direction of deterministic selection*, Proceedings of the National Academy of Sciences, doi:10.1073/pnas.1603693113.