Parallel Session

Mathematical Methods in Biology IV

LAGUERRE POLYNOMIAL SOLUTIONS OF A CLASS OF NONLINEAR REACTION DIFFUSION EQUATION AND ITS APPLICATIONS IN BIOLOGY

Burcu Gürbüz

burcu.gurbuz@uskudar.edu.tr

Üsküdar University

Joint work with Mehmet Sezer (Celal Bayar University).

Keywords: Nonlinear reaction-diffusion models, Fisher-Kolmogoroff equation, Laguerre collocation method.

In this study, we present a numerical scheme for the approximate solutions of a class of nonlinear reaction diffusion equation which arise in biological models. One of the classic case of a nonlinear reaction diffusion equation, Fisher-Kolmogoroff equation (EFK) is

$$\frac{\partial u}{\partial t} = ku(1-u) + D\frac{\partial^2 u}{\partial x^2}$$

where k and D are positive parameters. It was suggested by Fisher (1937) as a deterministic version of a stochastic model for the spatial spread of a favoured gene in a population. Furthermore, it is the natural extension of the logistic growth population model when the population disperses via linear diffusion. In addition to discussion of this biological model we present a numerical method is based on the Laguerre collocation method. The approximate solution of the problem in the truncated Laguerre series form is obtained by the present method. By substituting truncated Laguerre series solution into the problem and by using the matrix operations and the collocation points, the suggested scheme reduces the problem to an algebraic equation system. By solving this equation system, the unknown Laguerre coefficients can be computed. The accuracy and efficiency of the method is studied by comparing with other numerical methods when used to solve some numerical experiments.

References

[1] J. D. Murray. *Mathematical Biology 1: An Introduction*. Springer, Edition, Berlin, 2002.

[2] M. Ilati, M. Dehghan. (2017). *Direct local boundary integral equation method for numerical solution of extended Fisher-Kolmogorov equation*. Engineering with Computers 2017, 1–11.

[3] F. Liu, X. Zhao, B. Liu. (2017). *Fourier pseudo-spectral method for the extended Fisher-Kolmogorov equation in two dimensions*. Advances in Difference Equations 1, 94, 1–17.

Parallel Session

Mathematical Methods in Biology IV

AN EFFICIENT NUMERICAL ALGORITHM FOR AUTO-CONVOLUTION VOLTERRA INTEGRAL EQUATIONS

Chengming Huang

chengming_huang@hotmail.com

School of Mathematics and Statistics, Huazhong University of Science and Technology, Wuhan 430074, China

Joint work with Min Li (Huazhong University of Science and Technology).

Keywords: Volterra integral equation, Linear barycentric rational interpolation, Barycentric rational quadrature, Collocation method.

Volterra integral equations are frequently encountered in biological and physical models. In this talk, we consider the numerical simulation of auto-convolution Volterra integral equations based on a barycentric rational quadrature method. The main idea is applying a direct quadrature method based on linear barycentric rational interpolation to discretize such equations. A collocation method is used as the starting procedure. The boundedness and convergence of the numerical solution are investigated in detail. Some numerical experiments are carried out to confirm the theoretical results.

Acknowledgements: This work was supported by the National Natural Science Foundation of China (No. 11771163).

References

[1] Author. *Title of the book*. Publisher, Edition, City, Year.

[2] Authors. (year). *Title of the Paper*, Journal Volume, pp1-pp9.

Parallel Session

Mathematical Methods in Biology IV

POLYNOMIAL CHAOS EXPANSIONS FOR STABILITY, UNCERTAIN QUANTIFICATION AND SENSITIVITY ANALYSIS OF EQUILIBRIA OF UNCERTAIN DELAY DIFFERENTIAL EQUATIONS

Rossana Vermiglio

rossana.vermiglio@uniud.it

(Department of Mathematics, Computer Science and Physics, University of Udine, via delle Scienze, 206, Italy)

Joint work with A. Zamolo (Master Degree in Mathematics, University of Udine)

Keywords: Uncertain delay differential equation, Equilibria stability, Polynomial chaos, Uncertain quantification, Sensitivity analysis.

Delay differential equations have received great attention in mathematical modeling of population dynamics. They describe infinite-dimensional dynamical systems and, within this context, a first key question concerns the local stability of equilibria, which, according to the principle of linearized stability, can be inferred from the stability of the zero-solution of the linearized system. But in applications one has also to take into account the specification of model constants and parameters. In many situations, due to limitations in available experimental data, in measurement or identification of model constants, the input data cannot be exactly specified and, as a consequence, the analysis of the effect of data uncertainty on the stability of linear delay differential equations is a crucial question. The uncertain parameters are often modelled as random quantities in a suitable probabilistic framework. The Polynomial Chaos Expansions (PCEs) have been successfully applied to represent random variables [4], to quantify the effect of uncertainty and analyse the sensitivity in many applications [2]. The PC theory provides the basis for the definition of delay differential equations with uncertain parameters, here called uncertain delay differential equations [3], and for the Sobol sensitivity analysis of the stability indicator [5]. We propose a numerical approach which combines the spectral discretization of the infinitesimal generator [1] with non-intrusive methods [2]. Numerical results complete the talk.

References

[1] Breda D., Maset S. and Vermiglio R. *Stability of Linear Delay Differential Equations - A Numerical Approach with MATLAB*, SpringerBriefs in Control, Automation and Robotics, Springer-Verlag,1 edition, New York (2015).

[2] Le Maître O. P., Knio O. M. *Spectral Methods for Uncertainty Quantification*, Scientific Computation, Springer,1 edition, Netherlands (2010).

[3] Vermiglio R. (2017) *Polynomial Chaos expansions for the stability analysis of uncertain delay differential equations*. SIAM/ASA J. Uncertainty Quantification, 5(1), 278–303.

[4] Wiener N. (1938) *The Homogeneous Chaos*. American Journal of Mathematics 60(4), 897–936.

[5] Zamolo A. (2017) *Polynomial Chaos Expansions with applications to delay differential equations* Master's Thesis in Mathematics

Parallel Session

Mathematical Methods in Biology IV

PREDICTION AND PREDICTABILITY IN POPULATION BIOLOGY

Luís Mateus

lgmateus@fc.ul.pt

Centro de Matemática, Aplicações Fundamentais e Investigação Operacional, Universidade de Lisboa, Portugal

Joint work with Nico Stollenwerk (CMAF-CIO) and Maíra Aguiar (CMAF-CIO).

Keywords: Stochastic processes, Bayesian prediction.

To determine best predictors and quantify prediction uncertainties, we investigate an analytically solvable stochastic system from epidemiology for which the time dependent solution, the likelihood function and the Bayesian posterior can be explicitly calculated as functions of given data. We show analytical expressions for the prediction probability conditioned on best estimators of parameters versus prediction probability conditioned on data only, and marginalized over the parameters, observing that the prediction uncertainty is wider in the second case, as should be done in empirical studies. Though the concept becomes clear in the analytical study, the differences between prediction based on data directly and prediction based on best estimates of parameters is small due to the simplicity of the model. In a slightly more complex model which however already cannot be treated analytically, we clearly observe the expected large differences between the two predictions. [1]

References

[1] N. Stollenwerk, L. Mateus, F. Rocha, U. Skwara, P. Ghaffari, and M. Aguiar. (2015). *Prediction and Predictability in Population Biology: Noise and Chaos*, Math. Model. Nat. Phenom., Volume 10 **2**, 142–164.

Parallel Session

Mathematical Methods in Biology IV

ANIMAL MOVEMENT: SYMBOLIC DYNAMICS AND TOPOLOGICAL CLASSIFICATION

Carlos Correia Ramos

ccr@uevora.pt

CIMA, Research Centre for Mathematics and Applications, Department of Mathematics, University of Évora

Keywords: Topological dynamics, Markov chains, Symbolic dynamics, Animal movement, Complex motion.

We introduce a deterministic discrete dynamical system which is used to classify a variety of types of movements, either two dimensional or three dimensional. The dynamical system is defined by a one-parameter family of bimodal interval maps, through iteration. The characterization of the movements is obtained from the topological classification of the discrete dynamical system. Techniques from symbolic dynamics and topological Markov chains are applied.

Acknowledgements: This work has been partially supported by Centro de Investigação em Matemática e Aplicações (CIMA) da Universidade de Évora through the grant UID-MAT-04674-2013 of FCT - Fundação para a Ciência e a Tecnologia.

Parallel Session

Mathematical Methods in Biology IV

SUSTAINABILITY INDICATOR SYSTEM FOR ASSESSMENT OF DAIRY FARMS AT VALE DO TAQUARI-RS (BRAZIL)

Claus Haetinger

chaet@univates.br

Postgraduate Program in Environment and Development (M/D), Universidade do Vale do Taquari - Univates, ZIP 95914-014, Lajeado-RS (Brazil)

Joint work with Claudete Rempel, Edson Moacir Ahlert, Carlos Candido da Silva Cyrne, Maico Fernando Herrmann (Univates)

Keywords: sustainability, sustainability indicators, environmental modelling, milk production.

Environmental sustainability refers to a way of doing agriculture, seeking to ensure sustainable productivity in the long term, by means of the adoption of ecologically sound management practices, with agricultural property being an integrated system, aimed at optimizing resources, not only at levels of economic production, but mainly seeking the stability of the system and thus ecological sustainability.

Rural enterprises play an important role in the brazilian economic scenario, however inefficiency in management negatively affects their performance, which is one of the problems studied. Globalisation affects agribusiness and also the basis for primary production in the world. The need for professionalism naturally arises on dairy farms.

It is estimated that 12.5% of milk production (one million liter per day) of brazilian Rio Grande do Sul state is coming from the region named Vale do Taquari, involving nearly $9,000$ families and 36 municipalities, who find in this activity conditions to remain in rural areas. This work aims to contribute to the management of these properties, by focusing on environmental aspects, evaluating the matriz importance versus performance and other environmental modellings ([3], [4]).

That paper gives a proposition of a panel model of indicators that provides a better process for a management of rural properties [2]. In the managerial aspect, a comparative study was made between the regions of the brazilian Vale do Taquari and the spanish region of Galicia. After the data collection, it was possible to observe the non use of management indicators, which justified the elaboration of this proposal [1].

This study aims to present the development of an own instrument for analysis, called Indicators System for the Evaluation of the Sustainability of Milk Producing Properties, aiming at adapting the sustainability of the production system [6]. The system was developed in an Excel worksheet, using a set of 23 indicators in economic, social and environmental aspects, and a mobile version is currently being completed. The parameters analyzed in

the economic dimension were: productivity and income, income diversification, patrimonial evolution, basic services, education and training, technological evolution, enterprise management, family succession. For the social dimension the parameters were evaluated: physical, psychological, social, environmental and quality of life. In the environmental dimension, the following parameters were evaluated: waste, water, Permanent Preservation Area, Legal Reserve, pesticides and fertilizers, declivity, erosion, burning, land use ([5], [7], [8]). The system proved to be an adequate and sufficient method for field application to evaluate the impact of agricultural activities.

References

[1] C.C. da S. Cyrne, C. Haetinger, C. Rempel. *O (não) uso de indicadores de gestão em propriedades leiteiras do Vale do Taquari - RS*. Estudo & Debate 22(1) (2015), PP. 128-142.

[2] C.C. da S. Cyrne, C. Haetinger, C. Rempel, C. Scheneider. *Painel de indicadores em pequenas propriedades rurais*. Fronteiras: Journal of Social, Technological and Environmental Science 6(3) (2017), pp. 162-186.

[3] C.C. da S. Cyrne, C. Rempel, C. Haetinger, R.R. Eckhardt. *Avaliação da gestão ambiental em pequenas propriedades produtoras e leite no Vale do Taquari a partir do uso da matriz importância times desempenho*. Redes 20(2) (2015), pp. 176-194.

[4] C. Haetinger, C.C. da S. Cyrne, C. Rempel. *La gestion durable de l'environnement dans les exploitations porductrices de lait: une étude de cas dans le Vale do Taquari, RS, Brésil*. FACEF Pesquisa: Desenvolvimento e Gestão 19(3) (2016), pp. 319-328.

[5] D. Di Domenico, S.D. Kruger, S. Mazzioni, A. Zanin, M.B.D. Ludwig. *Índice de sustentabilidade ambiental na produção leiteira*. RACE 16(4) (2017), pp. 261-282.

[6] E.M. Ahlert, C. Haetinger, C. Rempel. *Sistema de indicadores para avaliação da sustentabilidade de propriedades produtoras de leite*. Estudo & Debate 24(2) (2017), pp. 23-49.

[7] M.C. Roloff, C. Rempel, R.R. Eckhardt. *Sustentabilidade ambiental de propriedades leiteiras do município de Paverama - RS*. Tecno-lógica 18(2) (2014), PP. 60-68.

[8] D. Pessi, C. Rempel, C. Haetinger. *Diagnóstico do uso da terra em Áreas de Preservação Permanente de pequenas propriedades rurais produtoras de leite no Vale do Taquari/RS - Brasil*. Espacios 38(22) (2017), pp. 7-16.