Tuesday, July 24th, 10:30 Room: C8.2.13

Parallel Session

Evolutionary Dynamics III

EVOLUTIONARY GAMES: NATURAL SELECTION OF STRATEGIES

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Keywords: Replicator dynamics, Natural selection, Strategy.

I model and study the process of natural selection between all possible mixed strategies in classical two-player two-strategy games. I derive and solve an equation that is a natural generalization of the Taylor-Jonker replicator equation that describes dynamics of pure strategy frequencies. I then investigate the evolution of not only frequencies of pure strategies but also of total distribution of mixed strategies. To solve these problems, I use the recently developed HKV (hidden keystone variable) method [1, 2]. I show that the process of natural selection of strategies for all games obeys the dynamical Principle of minimum of information gain. I also show a principle difference between mixed-strategies Hawk-Dove (HD) game and all other games (Prisoner's Dilemma, Harmony and Stag-Hunt games). Mathematically, the limit distribution of strategies is non-singular and the information gain tends to a finite value for HD-game, in contrast with all other games. Biologically, the process of natural selection in HD-game follows non-Darwinian "selection of everybody" but for all other games we observe Darwinian "selection of the fittest".

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Parallel Session

Evolutionary Dynamics III

SOCIAL DILEMMAS IN SIMPLE EVOLUTIONARY GAMES

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Keywords: Evolutionary game theory, Social dilemmas, Stochastic dynamics, Fixing probabilities, Games on graphs.

One of the fundamental problems in evolutionary biology is to understand mechanisms promoting altruistic behavior in animal and human populations. We address it within three simple models of evolutionary game theory.

In the Stag-hunt game, we are concerned with fixing probabilities of two absorbing states, cooperation and defection. We formulate two simple and intuitive criteria for evolutionary stability of pure Nash equilibria in finite populations. In particular, we show that the 1/3 law of evolutionary games [1] follows from a more general mean-potential law [2].

In the Snowdrift game, we show that a method of matching individuals has a strong influence on the long-run behavior of evolving populations. In particular, we show that lattice structure is always beneficial to cooperation as compared to random-matching models [3] but not to replicator dynamics [4].

In the Prisoner's Dilemma game played on the random scale-free Barabási-Albert graphs, we introduce a cost of maintaining a link between interacting players [5]. We show that when the cost increases, the population of players undergoes a sharp transition from an ordered state, where almost all players cooperate, to a state in which both cooperators and defectors coexist [6]. At the critical cost, the population oscillates in time between these two states.

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References

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Parallel Session

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UNINVADABLE STRATEGIES FOR BIOTROPHIC PATHOGENS, FROM DYNAMIC GAMES TO ADAPTIVE DYNAMICS

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Joint work with Ivan Yegorov (U. of Melbourne, Australia), Julien Guégan, Pierre Bernhard (Inria, France), Ludovic Mailleret, Fabien Halkett (INRA, France).

Keywords: Uninvadable strategy, Differential game, Adaptive dynamics, Resource allocation, Biotrophic pathogens.

In this work, we study the competition of two biotrophic fungal cohorts within a common host plant. Their identical dynamics are described by nonlinear ordinary differential equations representing the evolution of the average mycellium size M_i of each, and driven by their potentially differing resource allocation strategy over time $u_i(t)$. Indeed, biotrophic pathogens uptake resource from living host plant tissues and opt for its allocation to mycellium growth, spore (S_i) production, or a mix of both:

$$\begin{aligned} \frac{dM_i(t)}{dt} &= (1-u_i(t))\, f_i(M_1(t), M_2(t)) - g(M_i(t)) \\ \frac{dS_i(t)}{dt} &= u_i(t)\, f_i(M_1(t), M_2(t)) \end{aligned}$$

Each cohort's fitness is then defined as the per capita expected spore production over a season of duration T, $J_i(u_1(.), u_2(.))$. In an uninvadability context, both cohorts then stand in a zero-sum game where they try to maximize/minimize $J_1(.) - J_2(.)$ [3], in an approach complementary to [4]. The related Cauchy problem for the Hamilton-Jacobi-Isaacs equation is investigated through analytical and numerical approaches to obtain the solution of the zero-sum state-feedback game defined above. A first property that we obtained is that, if $M_1(0) = M_2(0)$, the Nash equilibrium corresponds to identical strategies for both cohorts. These strategies most classically consist in increasing the mycellium size upto some fixed level (with $u_i(t) = 0$), then keeping the mycellium constant at that level while producing spores (with $0 < u_i(t) < 1$), and finally only producing spores (with $u_i(t) = 1$) until the end of the season. The obtained solution was then compared to two complementary approaches: the one obtained from single-cohort optimal resource allocation [1] and the one stemming from adaptive dynamics. We noted that the solution of the optimal approach gives a similar type of solution, except that the mycellium level at which the solution settles in the intermediate phase is lower. We then noted from the generic form of the solutions obtained in both the optimal and game solutions that two scalar traits can define these allocation strategies: the level at which the mycellium settles during the intermediate phase, and the time before the end of the season at which the sporulation-only phase starts. Computing the equilibrium, canonical equation and stochastic adaptive dynamics for the latter problem [2], we obtained results in accordance with what the solution of the game yielded, which validates the approach.

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Parallel Session

Evolutionary Dynamics III

SEX-RATIO CONFLICT DISRUPTS COLONY GROWTH IN ANNUAL HAPLODIPLOID SOCIAL INSECTS

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Joint work with Lutz Fromhage (University of Jyväskylä), Mauricio Gonzalez-Forero (University of St Andrews) and Laurent Lehmann (University of Lausanne).

Keywords: Dynamic game theory, Optimal control theory, Social insects, Life-history evolution, Sex allocation.

The optimal life-history strategy for an annual eusocial colony generally proceeds in two phases: in the ergonomic phase, all effort is directed towards producing workers; in the reproductive phase, all effort is directed towards producing sexual offspring. The switch from the ergonomic to the reproductive phase is a key life-history event in annual eusocial insects. We show that the conflict over sex allocation between the queen and the workers gives rise to a suboptimal pattern of colony growth, characterized by a premature switch from the ergonomic to the reproductive phase. In the reproductive phase, males are expected to be produced before females (protandry) due to sexual selection if reproductives can disperse over a substantial period of time. We show that the time of dispersal affects the outcome of the sex allocation conflict. If reproductives disperse at the end of the season then the queen wins the sex allocation conflict even if the workers have full control over the developmental fate of the female brood. If reproductives disperse continuously throughout the season, then strongly male-biased sex ratio can evolve, but the queen does not win the sex allocation conflict if she is not in full control over the resource allocation traits. Our study highlights the time-dependent nature of within-colony conflict and the importance of considering the ontogenetic context of colony growth when studying sex allocation conflict in social insects. Furthermore, we show how classical static models of resource allocation conflict may be inadequate in predicting the outcome of the sex allocation conflict, since they are not able to capture the decoupling of resource allocation trade-offs for different phases of colony ontogeny. Our model provides predictions for determining the party in control of the resource allocation decisions in annual eusocial insects.

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RESIDENT-INVADER DYNAMICS OF SIMILAR STRATEGIES IN NOISY ENVIRONMENTS

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Joint work with Stefan A.H. Geritz (University of Helsinki) and Mats Gyllenberg (University of Helsinki).

Keywords: Adaptive dynamics, Environmental driver, Continuous-time model, Outcome of an invasion event.

We consider the resident-invader dynamics of similar strategies in a noisy environment. We focus on a class of continuous time models for structured populations and multi-dimensional strategies. We classify all generic population dynamical outcomes of an invasion event when the resident population size is *log-bounded on average*. The theoretical and numerical results show that the sum of the resident and invader population sizes varies stochastically and dramatically in fast time, while the proportion of the invader population size changes deterministically and monotonically in slow time. We show that invasion without back-invasion (i.e. when the roles of resident and invader are reversed) implies substitution, while mutual invasion leads to the coexistence of resident and invader at a unique population dynamical attractor. Our results generalise earlier results for resident dynamics with a stable equilibrium ([1], [2]) to resident dynamics with a stochastic (or deterministic) environmental driver.

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Parallel Session

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AN EXTENSION OF THE CLASSIFICATION OF EVOLUTIONARILY SINGULAR STRATEGIES IN ADAPTIVE DYNAMICS

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Keywords: Adaptive dynamics, Evolutionary singularity, Evolutionary Stable Strategies, Evolutionary branching, Nested model.

The basic framework of Adaptive dynamics assumes an invasion fitness that is differentiable twice as a function of both the resident and the invader trait. Motivated by nested models of infectious disease dynamics we consider an extended framework in which the selection gradient exists (so the definition of evolutionary singularities extends *verbatim*), but where invasion fitness may lack the smoothness necessary for the classification à la Geritz et al. [1]. We present the classification of evolutionarily singular strategies with respect to convergence stability and invadability and determine the condition for existence of nearby dimorphisms. The extended setting of Adaptive dynamics allows for a new type of evolutionary singularity: a so called one-sided ESS that is invadable by mutant strategies on one side of the singularity but uninvadable by mutants on the other side. We discuss possible evolutionary scenarios nearby one-sided ESSs and conclude by applying the extended framework to nested models of infectious disease dynamics.

The talk is based on joint work with Odo Diekmann [2].

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