MINISYMPOSIUM

PARTIAL DIFFERENTIAL EQUATION MODELS IN ECOLOGY AND EVOLUTION

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Minisymposium Keywords: Dispersal, diffusion, population structure, random genetic drift

Partial Differential Equations (PDEs) enjoy a long tradition in the modelling of population dynamics, and in particular of ecological and evolutionary dynamics.

In ecology, PDEs have been used extensively to describe the effects of dispersal on population dynamics by reaction-diffusion equations. Also in population genetics, there is a long history of using PDEs to describe the interaction of migration and selection on the evolutionary dynamics of a spatially structured population. There is also another line of research, dating back to the 1930ies, in which stochastic processes are approximated by reaction-diffusion equations to explore the effects of genetic drift in combination with selection and mutation on the evolutionary dynamics of a single population. Finally, PDE and integro-differential equations have been used to describe the evolutionary dynamics of a continuum of genetic types within a population.

While these problems have been mostly addressed on their own and gave rise to quite separate fields of biomathematical research, there are many situations where the combined interaction of population structure (typically spatio-temporal) with evolutionary forces will play an important and distinctive role in the way populations evolve. In particular, cross-fertilization between these areas will be extremely useful to facilitate diffusion of mathematical methods between them.

The aim of this minisymposium is to bring together recent research that has been tackling PDE models at the interface of ecology and evolution.

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CONTINUOUS APPROXIMATIONS OF FINITE-POPULATION STOCHASTIC MODELS: THE PDE VIEWPOINT

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Joint work with Fabio A. C. C. Chalub (Universidade Nova de Lisboa)

Keywords: Wright-Fisher process, Markov chains, Weak formulation, Conservation laws.

Using the Wright-Fisher (WF) process as a paradigm, we will show how to construct a 'discrete weak-formulation' for discrete-time, discrete Markov Chains (DTDMC). This discrete, weak formulation will then be used to obtain a degenerated parabolic PDE in the ultra-weak formulation, when considering the limit of infinite population. Depending on the different scaling assumptions, this PDE can be purely diffusive, convective-diffusive, or hyperbolic, and in the particular case of the WF process, it will be measure-valued. We also show that the hyperbolic PDE is equivalent to the Replicator Dynamics.

The ultra weak formulation is very natural and for models without mutation it provides the correct continuous counterpart of the conservation laws satisfied by the discrete process. This approach provides a unified framework to derive several PDE approximations to DT-DMCs which might describe either evolutionary, ecological o epidemiological dynamics, and which might include additional features, as population and spatial structuring for instance.

These results yield further confirmation what is now a common wisdom in population dynamics: PDEs can provide useful and powerful approximations to the dynamics of large populations in many different contexts. From this observation, we will then provide a brief review on the PDE modelling of evolutionary and ecological population dynamics, focusing on the interplay between evolutionary and spatial-temporal structuring dynamics.

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DYNAMICS OF AN ASEXUAL POPULATION FACING CLIMATE CHANGE

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Keywords: Structured population, Nonlocal reaction-diffusion equation, Propagation.

We consider an asexual population structured by a phenotypic trait and a spacial variable. The model is a parabolic partial differential equation with a nonlocal (in the phenotypic variable) competition term. We show that propagative fronts exist for that model, and explicit their speed. We can then push the analysis further: describe the dynamics of population in a 2D space with a heterogeneous landscape. We describe then the dynamics of the range of the population. This approach is interesting for its simplicity (the dynamics of the range is given by an explicit speed of the interface) and because the range of the population can be related to presence-absence maps that are used by field biologists. We illustrate applications of these results: they can provide an interesting insight to investigate the impact of natural reserves or mountains

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CLINES WITH PARTIAL PANMIXIA ACROSS A GEOGRAPHICAL BARRIER IN AN UNBOUNDED UNIDIMENSIONAL HABITAT

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Joint work with Yantao Wang (Southern University of Science and Technology, China) and Shuqing Chai (Southern University of Science and Technology, China)

Keywords: Geographical structure, Spatial structure, Subdivided populations, Migration, Long-distance migration.

We investigate a single-locus diallelic model for viability selection, local migration, and long-distance migration (approximated by partial global panmixia) in a linear unbounded habitat with a geographical barrier that may not be symmetric. In particular, we aim to understand the difference that the geographical barrier makes to the existence, multiplicity, and the shape of clines with various dominance.

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ASYMPTOTIC BEHAVIOR OF SOME SELECTION-MUTATION EQUATIONS

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Keywords: Selection, Mutation, Structured populations.

Selection mutation equations are mathematical models of Darwinian evolution. In this talk we will analyze the behavior for large time and small mutation rate of a selection- mutation-competition model for a population structured with respect to a phenotypic trait. We are interested in the interplay between the time variable t and the rate ε of mutations. We will show that depending on $\alpha > 0$ the limit $\varepsilon \to 0$ with $t = \varepsilon^{-\alpha}$ can lead to population number densities which are either Gaussian like (when α is small) or Cauchy-like (when α is large). So, on the one hand we analyze transient dynamics, which could be important in many biological situations (invasions, infections...) and, on the other hand, we determine the asymptotic profile of the densities, which shows heavy tails. This could also be relevant for the survival of the population under environmental changes.