MINISYMPOSIUM

THE INTERPLAY BETWEEN SHORT- AND LONG-RANGE INTERACTIONS IN BIOLOGY

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Minisymposium Keywords: Interacting particle systems, Agent-based modelling, Continuum limits.

Many biological systems involve a large set of individuals interacting via a combination of short- and long-range forces. Examples include bacterial or cellular chemotaxis. In cellular chemotaxis, cells may experience short-range excluded volume repulsion and a long-range chemotactic drift secreted by a different type of cells. This session will bring together experts working on a variety of biological problems where the interplay between short- and long-range interactions between individuals play an important role in the system dynamics. We will discuss different mathematical tools, from agent-based modelling to mean-field methods, to tackle such problems.

We contacted several groups that have been deriving these models with different applications in biological contexts:

- Zebra Fish patterning models: agent-based models have been recently proposed by a group in Brown University able to obtain the different patterns observed in nature. We will have a talk from the recent PhD student in this topic now postdoc at the MBI, Alexandria Volkening. Also, Markus Schmidtchen will discuss properties and simulations based on mean field equations obtained from coarse graining of these agent based models.
- Cell Sorting and Zebra Fish Neural Crest: these agent-based models have also been used to model adhesion and volume constraints to a variety of applications such as cell sorting and neural crest primordium. The group in Torino has worked in this direction in the last years. We have invited Marco Scianna.
- A connection between these nonlocal models and game theoretical interpretations both at the agent-based as well as the mean-field level has been explored by a group in Saint Andrews. Tomasso Lorenzi is invited in this topic.

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AGENT-BASED MODELS OF PATTERN FORMATION ON THE SKIN OF ZEBRAFISH

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Keywords: Zebrafish, Self-organization, Agent-based model, Pattern formation.

Wild-type zebrafish feature black and yellow stripes across their body and fins, but mutations display a range of altered patterns, including spots and labyrinth curves. These diverse patterns form due to the self-organizing interactions of pigment cells, which sort out through movement, birth, competition, and transitions in shape. Cells regulate each others' behavior on the growing skin by communicating both locally and at long range through dendritic extensions. Working closely with the biological data, we develop an agent-based model of this pattern formation, coupling deterministic migration by ODEs with stochastic rules for updating population size on growing domains. Our model proposes how a combination of short- and long-range interactions between cells is able to robustly produce stripes, and it suggests cell behaviors that may be altered to produce patterns on mutations and close relatives of zebrafish. We also explore stripe formation on the tailfin, where bone rays and epithelial growth may help direct pigment cell placement.

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PATTERN FORMATION IN NON-LOCAL SYSTEMS WITH CROSS-DIFFUSION

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Keywords: Pattern formation, Segregation phenomena, Nonlocal interactions, Zebrafish.

Multi-agent systems in nature oftentimes exhibit emergent behaviour, *i.e.* the formation of patterns in the absence of a leader or external stimuli such as light or food sources. We present a non-local two-species cross-interaction system of partial differential equations with cross-diffusion and explore its long-time behaviour. We observe a rich zoology of behaviours exhibiting phenomena such as mixing and/or segregation of both species and the formation of travelling pulses.

One of the most fascinating real world applications of this model are zebrafish with their black and yellow pigment cells whose interspecific and intraspecific interactions lead to the characteristic stripe pattern formation.

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ADHESION AND VOLUME CONSTRAINTS VIA NONLOCAL INTERACTIONS DETERMINE CELL ORGANISATION AND MIGRATION PROFILES

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Keywords: Interaction potentials, H-stability, Cell-cell interactions, Cell sorting.

The description of the cell spatial pattern and characteristic distances is fundamental in a wide range of physio-pathological biological phenomena, form morphogenesis to cancer growth. Discrete particle models are widely used in this field, since they are focused on the cell-level of abstraction and are able to preserve the identity of single individuals reproducing their behavior. In particular, a fundamental role in determining the usefulness and the realism of a particle mathematical approach is played by the choice of the intercellular pairwise interaction kernel and by the estimate of its parameters. We are going to show in this talk how the concept of H-stability, deriving from statistical mechanics, can have important implications in this respect. It in fact allows to a priori predict the stable configuration of a cell system for any given interaction kernel and relative coefficient setting. The proposed analytical results are of particular relevance also from an applicative perspective, as illustrated in the paper by series of representative simulations dealing with cell sorting phenomena and zebrafish embryonic development.

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CONTINUUM AND DISCRETE NONLOCAL MODELS OF SPATIAL EVOLUTIONARY GAMES

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Keywords: Nonlocal models, Spatial evolutionary games, Prisoner's dilemma, Agent-based models, Partial integro-differential equations.

Recently increasing attention has been given to mathematical models of spatial evolutionary games which explicitly incorporate the movement of the individuals playing the game. However, the results of most published studies are based only on computational simulations of agent-based models, which are usually inaccessible to analytical techniques. In this talk we will present a new modelling framework whereby numerical simulations of nonlocal agent-based models can be combined with the analysis of corresponding continuum models formulated in terms of partial integro-differential equations. Compared with research approaches relying solely on the computational simulation of agent-based models, such a framework allows one to achieve more robust and precise biological conclusions. As an example, we focus on the case of a spatially explicit prisoner's dilemma model. The analytical and numerical results that we have obtained in this case illustrate how nonlocal interactions can bring about self-generated spatial patterns which create favourable conditions for the coexistence of cooperators and defectors in situations where the two strategies cannot coexist in the presence of either global or local interactions.