

Parallel Session  
**Epidemiology XI**

**MATHEMATICAL MODELING AND ANALYSIS OF THE  
TRANSMISSION DYNAMICS OF TUBERCULOSIS AND  
LYMPHATIC *FILARIASIS* CO-INFECTION**

EGBERANMWEN BARRY SUNDAY IYARE

barriyare@yahoo.com

Department of Mathematical and physical Science, College of Basic and Applied Sciences, Samuel Adegboyega University, KM 1, Ogwa-Ehor Road, Ogwa, PMB 0001, Edo, 300001, Nigeria

Joint work with Dr. Daniel Okuonghae (University of Benin, Nigeria), Prof. Francis E.U. Osagiede (University of Benin, Nigeria).

*Keywords:* Lymphatic *filariasis*, Tuberculosis, Co-infection.

A model for the transmission dynamics of Tuberculosis - Lymphatic *filariasis* co-infection in a population where they are both endemic is developed in order to investigate the impact of lymphatic *filariasis* on the transmission dynamics of tuberculosis. We analysed the single models steady states and then investigate the existence and stability of equilibria. It was found that both the lymphatic *filariasis*-only and co-infection models exhibit backward bifurcations even when the disease-induced death was negligible. And that lymphatic *filariasis* may be associated with an increased risk of tuberculosis infection. Optimal control theory was then applied to determine the optimal strategy for curtailing the spread of both diseases. Our result shows that the optimal strategy for controlling the spread of both diseases is the strategy combining all the time dependent controls.

Parallel Session  
Epidemiology XI

**DYNAMICAL MODELS OF TUBERCULOSIS  
TRANSMISSION AND OPTIMAL TREATMENT  
STRATEGIES IN THE REPUBLIC OF KOREA AND  
PHILIPPINES**

EUNOK JUNG

junge@konkuk.ac.kr

Department of Mathematics, Konkuk University, Seoul, Republic of Korea

Joint work with Aurelio A. de los Reyes V (University of the Philippines Diliman), Soyoung Kim (Konkuk University) and Sunhwa Choi (National Cancer Center Korea).

*Keywords:* Tuberculosis, Epidemic model, Optimal control theory.

In this talk, we will present several mathematical models of tuberculosis (TB) based on the reported data in the Republic of Korea and Philippines, and also propose the optimal treatment strategies depending on the various scenarios in each country. Korea has ranked the highest TB incidence among members of the Organization for Economic Cooperation and Development (OECD). TB is the sixth leading cause of morbidity and mortality in the Philippines. The least-square curve fitting have been used for best fitting the parameters in our models to the observed data. To determine the optimal intervention strategy which is reducing the number of exposed and infectious individuals and the cost of control measures, optimal control theory was used [1]. Important issues has been addressed from our research: implementing the smoking controls, not with TB controls, can derive significant reduction of the incidence of TB transmission [3]. We suggested the rearrangement of the Korean government TB budget based on optimal treatment strategies from modeling [2]. Finally, in the Philippines enhancing active finding control is a significant control factor to curtail the spread of TB [4].

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Parallel Session  
Epidemiology XI

## MATHEMATICAL MODELING OF SEX DIFFERENCES IN TB EPIDEMIOLOGY

TATIANA E. SANNIKOVA

te\_san@yahoo.com

Institute of Numerical Mathematics, Russian Academy of Sciences, Gubkina,8, Moscow, Russia

*Keywords:* Epidemiology, Tuberculosis.

Sexual inequality in death and diseases is a profoundly studied topic in such fields like demography, sociology and public health. In general, men are more prone to infectious disease and experience a shorter lifespan. But, in case of tuberculosis (TB) male-to-female ratio varies world widely from 1 in Thailand to 4.7 in Armenia with a mean value of 1.9. It is obvious that such a variety cannot be explained only by physiological differences in vulnerability to infections; socio-cultural norms and individual health-related behavior are very influential in revealing new cases and treating confirmed diseases.

Sex ratio in TB cases in different districts of Moscow city has been analysed for the period from 1999 to 2014 . The mathematical model of TB epidemiology in two-sex population has been developed and fitted to the data. The estimated values of the parameters make it possible to evaluate impact of social, behavioral and biological factors . Since the rates of TB detection and the rates of recovery in treated females were higher than in males it can be concluded that female behavior can be characterized as more health-related than male behavior which is in compliance with practitioners' observations. At the same time transmission rates are lower in females than in males which can be explained by the fact that females have less contacts with socially ill-adapted individuals who form a high risk group for TB.

The results obtained demonstrate that a behavioral factor is one of the most significant in understanding sex differences in TB epidemiology and should be taken into consideration during the elaboration of TB control strategies.

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Parallel Session  
Epidemiology XI

## STABILITY OF A FRACTIONAL TUBERCULOSIS MODEL

DELFIN F. M. TORRES

delfim@ua.pt

CIDMA, Department of Mathematics, University of Aveiro, 3810-193 Aveiro, Portugal

Joint work with Weronika Wojtak and Cristiana J. Silva

*Keywords:* Tuberculosis, Non-integer order differentiation, Stability.

We propose a Caputo type fractional-order mathematical model for the transmission dynamics of tuberculosis (TB). Existence of equilibrium points is investigated and the uniform asymptotic stability of the unique endemic equilibrium is proved, via a suitable Lyapunov function, for any non-integer order of differentiation. Our analytical results were complemented by numerical simulations in *Matlab* illustrating the stability result. It is shown that the proposed fractional order model provides richer and more flexible results when compared with the corresponding classical (integer-order) TB model.

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Parallel Session  
Epidemiology XI

## NON-INTEGER ORDER ANALYSIS OF THE IMPACT OF DIABETES AND RESISTANT STRAINS IN A MODEL FOR TB INFECTION

CARLA PINTO

cap@isep.ipp.pt

School of Engineering, Polytechnic of Porto

Joint work with Ana R.M. Carvalho (Faculty of Sciences, University of Porto).

*Keywords:* Tuberculosis, Diabetes, Coinfection, Multi-drug resistance, Non-integer order model.

We study the impact of diabetes and multi-drug resistant strains in a non-integer order model for tuberculosis (TB) infection in a community. We compute the reproduction number,  $R_0$ , of the model and analyse its behaviour numerically for variation of epidemiologically relevant parameters. Namely, the increased susceptibility to TB due to diabetes, the diabetes recruitment rate, and the increased progression of non-diabetics TB infectious to diabetic TB infectious individuals, due to their active TB status. We have proven the global stability of the disease-free equilibrium for specific conditions, related with exogeneous and endogeneous reinfections, and relapse of recovered individuals. Numerical simulations of the model for the above mentioned parameters confirm the dynamics predicted by the value of  $R_0$ . For  $R_0 < 1$  the disease-free equilibrium is stable, and it becomes unstable for  $R_0 > 1$ . The sensitivity indexes of  $R_0$  are computed and discussed. The order of the fractional derivative adds more information about the complexity of the dynamics of the proposed model and may help distinguishing dynamical traits in distinct TB patients [1].

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